MATH 147 QUIZ 5 SOLUTIONS

1. Minimize $f(x, y, z) = x^2 + y^2 + z^2$ subject to the constraint $x + y + z = 1$. (5 Points)

   We solve using Lagrange multipliers. We have $\nabla f = (2x, 2y, 2z)$ and $\nabla g = (1, 1, 1)$, giving us the system of equations

$$\begin{cases} 2x = \lambda \\ 2y = \lambda \\ 2z = \lambda \\ x + y + z = 1. \end{cases}$$

   Combining the first three equations gives us $x = y = z$, and upon substitution into the last equation, we get $x = 1/3$ and so on. This gives us a value of $f(1/3, 1/3, 1/3) = 1/3$. This is a minimum: there is no max, as something with negative values can grow much larger, such as $f(100, -100, 0) = 20000$.

2. Find the absolute maximum and minimum values of $4xy$ subject to $\frac{x^2}{9} + \frac{y^2}{16} = 1$. (5 points)

   We again use lagrange multipliers. We get the system:

$$\begin{cases} 4y = 2\lambda x/9 \\ 4x = \lambda y/8 \\ \frac{x^2}{9} + \frac{y^2}{16} = 1. \end{cases}$$

   We can multiply the first equation by x and the second equation by y to get

$$4xy = \lambda x^2/18 \text{ and } 4xy = \lambda y^2/32.$$

   Setting them equal gives us $\lambda x^2/18 = \lambda y^2/32$ or equivalently $y^2/16 = x^2/9$. We can substitute these into the third equation to get $2x^2/9 = 1$ and $y^2/8 = 1$, giving us the critical points $x = \pm 3/\sqrt{2}$ and $y = \pm 2\sqrt{2}$. Just looking at the positive points, we get $f(3/\sqrt{2}, 2\sqrt{2}) = 24$. The signs in the critical point will only change the sign of the answer, so we conclude $(3/\sqrt{2}, 2\sqrt{2})$ and $(-3/\sqrt{2}, -2\sqrt{2})$ are the places we hit a maximum, with a value of 24. On the other hand, at $(3/\sqrt{2}, -2\sqrt{2})$ and $(-3/\sqrt{2}, 2\sqrt{2})$ we hit an absolute minimum of -24.